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Phase Calibration With Fast Switching: Implications for Instrumental Phase Stability

Larry R. D'Addario
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Consider interferometric observations of a target source at frequency f that are rapidly interspersed with observations of a nearby unresolved[1] calibrator at frequency f_c. The raw interferometer phase measurements for each elementary observation are then

$$\begin{aligned}\theta_{\text{target}} =& \phi_t + \phi_i(f) + 2\pi f\tau \\ \theta_{\text{cal}} =& \phi_i(f_c) + 2\pi f_c\tau'\end{aligned} \tag{1}$$

where ϕ_t is the intrinsic visibility phase of the target source; ϕ_i is the instrumental phase at the given frequency; and τ, τ' are the atmospheric delays during the target and calibrator observations, respectively. Here thermal noise has been neglected, and all of the phase values are assumed constant during each observation. Scaling the second measurement by f/f_c and subtracting allows much of the atmospheric phase effect to be removed[2]:

$$\phi_{\text{corrected}} = \phi_t + \phi_i(f) - (f/f_c)\phi_i(f_c) + 2\pi f\,\delta\tau \tag{2}$$

where $\delta\tau = \tau - \tau'$ is the change in atmospheric delay between target and calibrator observations. This result contains the desired target source visibility and the difference in scaled instrumental phase between the two frequencies, along with the residual atmospheric delay fluctuation.

To determine the instrumental phase difference between the two frequencies, we must carry out a separate observation in which a calibrator is observed at both frequencies; we call this the instrumental sequence, and we call the measurements in (1) the target sequence. The calibrator for the instrumental sequence or may not be the same as the one for the target sequence, but again we assume it to be unresolved. The instrumental sequence observations yield

$$\begin{aligned}\theta_{\text{cal1}}(f) =& \phi_{i1}(f) + 2\pi f\tau_1 \\ \theta_{\text{cal1}}(f_c) =& \phi_{i1}(f_c) + 2\pi f_c\tau_1'\end{aligned} \tag{3}$$

where subscript 1 distinguishes the values obtained in this observation from the similar values obtained during the target sequence. Scaling and subtracting gives

$$\phi_{\text{calibration}} = \phi_{i1}(f) - (f/f_c)\phi_{i1}(f_c) + 2\pi f\,\delta\tau_1, \tag{4}$$

and subtracting this from (2) gives

$$\phi_{\text{calibrated}} = \phi_t + \Delta\phi_i(f) - (f/f_c)\,\Delta\phi_i(f_c) + 2\pi f\,(\delta\tau - \delta\tau_1) \tag{5}$$

where $\Delta\phi_i = \phi_i - \phi_{i1}$ is the change in instrumental phase at the given frequency between the target sequence and the instrumental sequence. This result contains the desired target visibility phase plus residuals due to instrumental phase drift and uncorrected atmospheric delay fluctuation.

[1] The calibrator need not actually be unresolved, but its intrinsic visibility at f_c should be known. We take that phase to be zero for simplicity.

[2] This linear scaling with the frequency ratio is accurate if the atmosphere is non-dispersive, but at sub-millimeter wavelengths dispersion is often significant. In practice, more sophisticated scaling based on estimates of the group delay ratio can be used.

Application to ALMA

The ALMA telescope is designed to allow the target sequence to be carried out with a cycle time as short as about 10 sec, and we expect that the cycle time will typically be less than 20 sec. For example, a cycle might consist of 15 sec on target, 2 sec on calibrator, and two 1.5-sec transitions between the two. On such time scales, it is important that the instrumental phase variation be negligible compared with the atmospheric phase fluctuation. Within each elementary observation, fluctuations in atmospheric delay or instrumental phase will cause loss of coherence, but that is not considered here (see [1]). For the instrumental sequence, the cycle time will usually be shorter because very little antenna motion is needed (although the ALMA electronics specification still allows 1.5 sec for the frequency change) and because the source will often be strong enough to allow short integrations. Equations (1) and (3) show only one cycle, but in practice many such cycles will normally be executed and the results averaged. Therefore, the residual atmospheric delays $\delta\tau, \delta\tau_1$ should be treated statistically, characterized by their standard deviations $\sigma_{\delta\tau}$ and $\sigma_{\delta\tau 1}$. Because the target sequence and instrumental sequence may be substantially separated in time (perhaps many minutes) and also in angular distance on the sky, the two residuals may be taken to be independent.

Also because of the time between the target sequence and the instrumental sequence, the instrumental phase drifts $\Delta\phi_i(f)$ and $\Delta\phi_i(f_c)$ cannot be neglected. To keep the instrumental error well below the atmospheric error, we find from (5) that we should have

$$\Delta\phi_i(f) - (f/f_c)\,\Delta\phi_i(f_c) \ll 2\pi\, f\,\sqrt{\sigma_{\delta\tau}^2 + \sigma_{\delta\tau 1}^2}. \quad (6)$$

A portion of the instrumental phase variation at the two frequencies should be common, so that the subtraction on the LHS of (6) may be helpful, but the components that contribute most to any phase drift are separate at the two frequencies; therefore, for the purpose of making a worst-case estimate, we assume that the two phase drifts are unrelated and can have either sign. The scaling in (6) makes it appear that the instrumental effect is more sensitive to the drift at the lower of the frequencies. However, in practice the phase variation is larger at higher frequencies. If we express the variation in units of delay by defining $\Delta\tau_i(f) = \Delta\phi_i(f)/(2\pi f)$, (6) simplifies to

$$\Delta\tau_i(f) - \Delta\tau_i(f_c) \ll \sqrt{\sigma_{\delta\tau}^2 + \sigma_{\delta\tau 1}^2}. \quad (7)$$

If we now choose to set a specification limiting the standard deviation σ_i of $\Delta\tau_i$ to the same value for all frequencies, we should use

$$\sqrt{2}\,\sigma_i(\mathrm{max}) \ll \sqrt{\sigma_{\delta\tau}^2 + \sigma_{\delta\tau 1}^2}. \quad (8)$$

Note that here σ_i is the standard deviation of the instrumental drift at a time difference equal to the interval between the target sequence and the instrumental sequence. This will typically be at least several minutes, and for specification setting purposes we assume a maximum value of 1000 sec.

Determination of the atmospheric terms $\sigma_{\delta\tau}$ and $\sigma_{\delta\tau 1}$ is somewhat complicated. The residuals $\delta\tau$ and $\delta\tau_1$ depend on averaging over many cycles of their respective sequences, so the standard deviations depend on the duration of each sequence as well as on the autocorrelation function of the atmospheric delay τ. Furthermore, in practice the residuals can be made smaller by a more sophisticated procedure than simply subtracting the pairs of measurements for each cycle as indicated in (2) and (4). Especially for the target sequence, instead of obtaining a single value of θ_{target} by integrating over the available time, one could obtain a sequence of samples using shorter integrations, then correct each one separately using an interpolated value of θ_{cal} from the two adjacent calibrator observations. An analysis in which the residuals from such a process are estimated from the ALMA site test data will be the subject of a separate paper [2].

Under some circumstances, it is possible to execute the target sequence with $f_c = f$, in which case there is no need for the instrumental sequence. Then the specification (8) becomes

$$\sigma_i(\mathrm{max}) \ll \sigma_{\delta\tau}, \quad (9)$$

but now σ_i applies at a time interval equal to the target sequence cycle time, typically 20 sec, rather than the interval between the sequences, up to 1000 sec. The actual instrumental error is expected to be far smaller for such short intervals, so if (8) is satisfied then (9) is easily satisfied.

Use of Water Vapor Radiometers

If the atmospheric delay can be corrected via water vapor radiometry, then the need for rapidly interspersed calibrator observations might be eliminated, leading to higher observing efficiency (more time on the target). We then have only the elementary observations of $\theta_{\rm target}$ in (1) and $\theta_{\rm cal1}(f)$ in (3). Subtracting these leaves an atmospheric error term $2\pi f(\tau - \tau_1)$ rather than $2\pi f\,(\delta\tau - \delta\tau_1)$; that is, we are left with the difference in atmospheric phase between target and calibrator, rather than just the difference in the within-sequence fluctuations. In this scenario, the target and calibrator observations are separated in time by many minutes, and they may also be separated by a substantial angle on the sky, including a change in elevation.

Present water vapor radiometer (WVR) technology does not allow the total delays τ, τ_1 to be derived, but only the variation in delay about some unknown mean value [3]. The mean value varies with air mass and drifts with time, so that differential results become inaccurate when elevation changes significantly and for time intervals greater than a few minutes. Furthermore, in this memo τ, τ_1 measure the effect on the interferometric phase, so they are actually the differences in atmospheric delay between the paths to the two antennas of a baseline. With WVRs, this must be estimated by subtracting the results of separate instruments. For these reasons, it is difficult to rely on WVR measurements alone for removal of phase errors due to atmospheric delay. However, if astronomical phase calibration is abandoned entirely, relying only on self-calibration, then WVR measurements may be effective as the sole means of preventing loss of coherence from short-term atmospheric delay fluctuations.

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REFERENCES

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