A comparison of the ALMA OTF holography transmitter power to the power radiated at mm wavelengths by a human body

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The ALMA holography transmitter at the OTF radiates about 10 microwatts, into an antenna with a gain of 33 dBi. This radiation is confined to a very narrow bandwidth, of order 1 kHz or less.
This radiated power is small compared to the mm-wave thermal radiation from a human body.

The following notes were written and calculated using Mathcad.

Some physical constants and formulae:

Speed of light, m/s $c := 2.99792458 \cdot 10^{8}$

Planck constant, J.s $h := 6.6260755 \cdot 10^{-34}$

Boltzmann constant, J/K $k := 1.380658 \cdot 10^{-23}$

Stefan's constant, W/m^2/K^4 $\sigma := 5.67051 \cdot 10^{-8}$

Assumed radiating surface area of a human body, square meters: $A := 1$

Assumed temperature of human body: T=300 K

Holography frequency: $fh := 100$ GHz

Transmitter antenna gain $G := 33$ dBi

Planck radiation law.

Watts per Hz at frequency f(Hz) and temperature T (K).

$$Wp(f,T) := \frac{2 \cdot A \cdot h \cdot f^{3}}{c^{2} \cdot \left(e^{h \cdot \frac{f}{k \cdot T}} - 1 \right)} \cdot \pi$$

Stefan's law, total power radiated integrated over all frequencies:

$$Ws(T) := \sigma \cdot A \cdot T^{4}$$

Check:

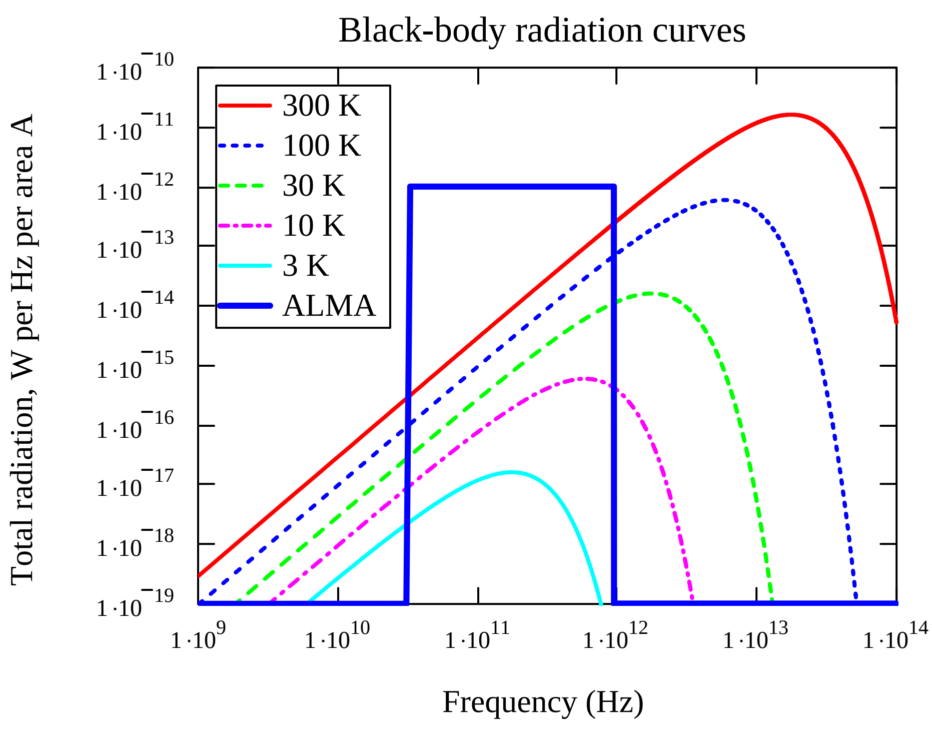
Total Thermal Power radiated: Example of $T := 30$

Comparison Planck with Stefan's law. At temperature T=30, between 0 and 10^15 Hz:

Planck integration: $\int_0^{10^{15}} Wp(f,T)\,df = 0.04593$ watts

Stefan's law: $\sigma \cdot A \cdot T^4 = 0.04593$ watts

So the integration of Planck's law is getting the right answer.
This validates the Mathcad Planck equation and integration.



QUESTION:

What is the 300 K thermal mm-wave power radiated over the spectrum covered by ALMA, from 31 GHz to 950 GHz?

$$\text{Pthermal} := \int_{31\cdot 10^{9}}^{950\cdot 10^{9}} \text{Wp}(f, 300)\, df \qquad \text{Pthermal} = 0.078 \quad \text{watts}$$

Compare this to the 10*10-6 watts from the transmitter.

$$\text{Ptx} := 10\cdot 10^{-6} \text{ watts} \qquad \frac{\text{Pthermal}}{\text{Ptx}} = 7.813 \times 10^{3}$$

If we apply the antenna gain of G= 33 dB to the transmitter power:

$$\text{ratio} := \frac{\text{Pthermal}}{\left(\text{Ptx}\cdot 10^{\frac{G}{10}}\right)} \qquad \text{ratio} = 3.916$$

So, even with the transmitter antenna beamed directly at,say, a fuel tank,
a person standing at the base of the tower radiates
mm-wave radiation ~4 times stronger than the holography transmitter.