Series Section Transmission Line Transformers

by

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Figure 1 illustrates two transmission line sections, of different characteristic impedance, in series and connecting a load impecance Z_L to a "generator" impedance Z_G. The transmission line sections match the two impedances, or transform the one impedance into the other. This arrangement can be called a series section transmission line transformer, or with the understanding that transmission lines are involved, a series section transformer.

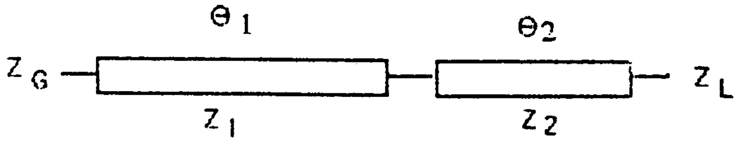

Figure The Series Section Transmission Line Transformer

The behavior of this arrangement can be described as that of two symmetrical four-terminal networks with real characteristic impedances connected in cascade.

The range of impedances that can be matched is easily deduced from R-X diagrams. Figure 2 illustrates an R-X diagram for a series section transformer.

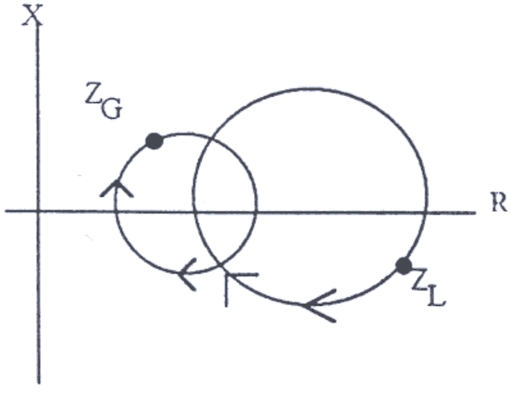

Figure 2. A Series section Transformer Matching Two Impedances

To be able to match the two impedances it is necessary that the SWR circles for the two lines to intersect.

Figure 3 illustrates the two extreme cases.

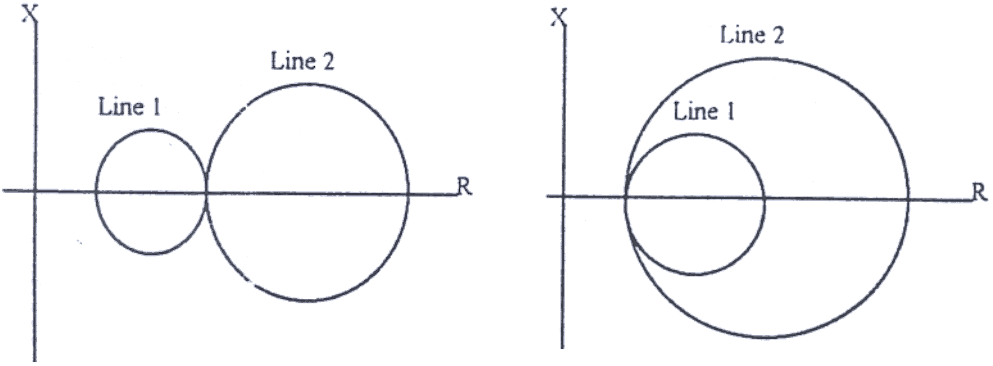

Figure 3. Limiting Cases for Matching Two Impedances.

For any specified SWR on, say, line 1, say S_1, the SWR on line 2, S_2, must fall between $\frac{Z_1}{Z_2} S_1$ and $\frac{Z_2}{Z_1} S_1$,

$$\frac{Z_1}{Z_2} S_1 \leq S_2 \leq \frac{Z_2}{Z_1} S_1,$$

where Z_2 is assumed greater than Z_1. (Otherwise, the inequalities are reversed.)

The Alternating Line Transformer

Perhaps the most familiar form of the series section transformer is the alternating line transformer, where two line sections of characteristic impedances Z_1 and Z_2 are used to convert a resistive impedance of Z_2 to a resistive impedance Z_1. Figure 4 illustrates the arrangement.

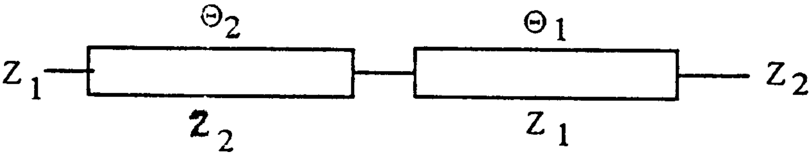

Figure 4. The Alternating Line Transformer.

For this arrangement, $\Theta_1 = \Theta_2 = \Theta$, and

$$\tan^2 \Theta = \frac{\left(\frac{Z_2}{Z_1}\right)^2 - \frac{Z_2}{Z_1}}{\left(\frac{Z_2}{Z_1}\right)^3 - 1}$$

This solution is most easily obtained by transforming each terminating impedance through the attached line to the center junction of the two lines (note that the line length or electrical angle will be negative for one of these transformations) and separately equating the real and imaginary parts of the resulting transformed impedances.

For lines of characteristic impedances of 50 and 75 Ohms, the electrical lengths are 29.334 degrees.

Note that in accordance with Figure 2, two solutions are possible depending on the sign selected for the square root(s). One solution will typically offer the shortest total line length, and this solution would ordinarily be selected for use.

The bandwidth of the simple alternating line transformer is about ±10 percent at an SWR of 1.07--comparable to that of a single section quarter wave transformer. Like the quarter wave transformer, the bandwidth of the alternating line transformer can be greatly increased by carrying out the impedance transformation in two steps centered on the mean impedance $\sqrt{Z_1 Z_2}$. Figure 5 illustrates the arrangement.

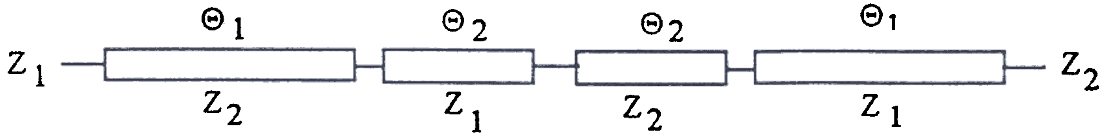

Figure 5. A Two Step Alternating Line Transfonner.

Values for the electrical line lengths can be obtained in the same way as for the single step alternating line transformer, with considerably more complex algebraic manipulations. The solutions are

$$\tan^2 \Theta_2 = \sqrt{k}\,\frac{k^{\frac{3}{2}} - 1}{k^{\frac{5}{2}} - 1} \qquad \tan \Theta_1 = \sqrt{k}\,\frac{k^{\frac{1}{2}} - 1}{k^{\frac{3}{2}}} \tan \Theta_2 ,$$

$$k = \frac{Z_2}{Z_1}$$

For a 50 and 75 Ohm system, the appropriate electrical line lengths are $\Theta_1 = 14.105$ degrees and $\Theta_2 = 37.386$ degrees.

An interesting and useful aspect of two step alternating line transformers is that the R-X diagram (or Smith chart) of the right half of the transformer in impedance coordinates is geometrically identical to that of the left half in admittance coordinates. The SWR circle for line 1 in impedance coordinates becomes the SWR circle for line 2 in admittance coordinates, etc. Consequently the diagram need only to be drawn once to determine graphically the electrical length of the four lines.

The same is true for the two lines--the right and left halves--of a single step alternating line transformer. As both lines have the same SWR, it is thus obvious that the electrical lengths of the two lines must be identical.

Generalized Series Section Transformers

The alternating line transformer can be generalized. See Figure 6.

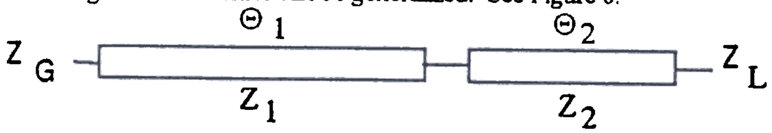

Figure 6. A Generalized Series Section Transformer with Resistive Input and Output.

If the input and output impedances are pure real (resistive) the appropriate electrical line lengths are given by

$$\tan\Theta_1 = \sqrt{\frac{(k-b)(1-ak)}{(a-k)(1-kb)}}, \quad \tan\Theta_2 = \sqrt{\frac{(a-k)(k-b)}{(1-ak)(1-kb)}},$$

$$k = \frac{Z_1}{Z_2}, \quad a = \frac{Z_L Z_G}{Z_1 Z_2}, \quad b = \frac{Z_L}{Z_G}$$

For a solution to exist, the quantities under the radicals must be positive.

For the generalized cases, solutions may be possible with either of the two line segments of differing characteristic impedance serving as the input line. In such cases, the solution having the greatest SWRs will probably have the shortest total line length. The solution having the lowest SWRs may have the greatest bandwidth.

It does not appear that the general case where both load and generator impedances are complex has a useful closed form solution. However, Regier (3) has published a solution for the special case of a resistive generator impedance equal to the characteristic impedance of the line connected to the load. With the arrangement of Figure 7,

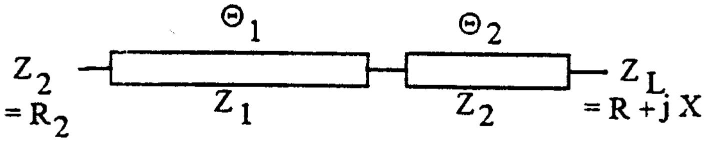

Figure 7. Regier's Special Case of a Series Section Transformer with a Complex Load

The solution is

$$Tan\Theta_2 = \frac{\left(Z_1 - \frac{Z_2 R}{Z_1}\right)\sqrt{\frac{k}{m-k}} + X}{R - Z_2 + Z_1 X\sqrt{\frac{k}{m-k}}}, \quad \tan\Theta_1 = \sqrt{\frac{k}{m-k}}$$

where

$$k = \frac{(R - Z_2)^2 + X^2}{RZ_2}, \quad \mathrm{m} = \frac{Z_1}{Z_2} - \frac{Z_2}{Z_1})^2$$

For a solution to exist, m>k, a condition satisfied if the SWRs are related as previously described

Regier's solution can be used to obtain an analytically designed generalized series section transformer, but one incorporating three line segments, the center segment being the sum of the line lengths solving Z_L to Z_2 and solving Z_G to Z_2 in the opposite direction. On an R-X diagram (or Smith chart) there are two SWR circles with Z_2 as the base impedance and one SWR circle with Z_1 as the basis. This last SWR circle must intersect the first two circles at and pass through impedance $Z_2 = R_2$.

While a useful general analytical solution for the series section transformer may not exist, a graphical solution on an R-X chart, or Smith Chart is straight forward. The Smith chart is probably the best choice in this case. The chart must be drawn twice, once normalized to the characteristic impedance of line 1 and once normalized to the characteristic impedance of line 2.

Figure 8 illustrates such a graphical solution.

Solutions to the generalized problem are also obtainable by numerical methods. Table 1 following is a Basic program for solving the series section transmission line transformer.

Table 1. A Numerical Solution to the Generalized Series Line Transformer

```
1 CLS
2'This program calculates the two solutions to the generalized two series
3'section transmission line transformer. First the SWR for each line is
4'calculated and from the SWR the center and radius of each SWR circle. The
5'two intersections of the two circles are determined, and the angles of the
6'common impedances calculated for the two characteristic impedances. The
7'differences between the angles of the two common impedances and the angles
8'of the input and output impedances are found and are the input and output
9'line lengths in angular measure.
10 print "    Solutions to the Generalized Two Series Section Transmission"
11 print "                   Line Transformer"
12 print "                        by"
13 print "               Albert E. Weller, WD8KBW"
14 print "                 January 28, 1995"
15 print:print
17 print "(Note that in the following, admittances may be substituted for impedances.)"
18 print
20 print "   Input or load resistance  "
22 input "           resistive part? ", RL
30 input "            reactive part? ", XL
40 input "   Characteristic impedance of input line? ", Zo2
42 print:print
50 print "   Output or generator impedance"
52 input "            resistive part? ", RG
60 input "             reactive part? ", XG
70 input "   Characteristic impedance of output line? ", Zo1
80 R=RL
```

```
90 X=XL
100 Z=Zo2
110 gosub 1000
120 SWR2=SWR
130 cen2=cen
140 rho2=rho
150 angZL=ang
155 ABSZL=ABSZ
160 R=RG
170 X=XG
180 Z=Zo1
190 gosub 1000
200 SWR1=SWR
201 'print SWR2, SWR1
210 cen1=cen
220 rho1=rho
230 angZG=ang
235 ABSZG=ABSZ
240 a=cen2-cen1
250 d=(rho1)^2-(rho2)^2+a^2
260 RC=cen1+d/(2*a)           'resistive part of the common impedance
262 L=((rho1)^2-(d/(2*a))^2)
265 if L<0 then goto 990
270 XCa=sqr(L)                 'reactive part of the common impedance
280 XCb=-sqr(L)                '   "    "   "   "    "        "
290 R=RC
300 X=XCa
310 Z=Zo2
320 gosub 1100
330 angZC2a=ang                'angle of the common impedance re line 2
340 Z=Zo1
350 gosub 1100
360 angZC1a=ang                'angle of the common impedance re line 1
370 X=XCb
380 gosub 1100
390 angZC1b=ang
400 Z=Zo2
410 gosub 1100
420 angZC2b=ang
430 theta2a=angZC2a-angZL
440 If theta2a<0 THEN theta2a=180 + theta2a
450 theta1a=angZG-angZC1a
460 if theta1a<0 then theta1a=180 + theta1a
470 theta2b= angZC2b-angZL
480 if theta2b<0 then theta2b=180 + theta2b
490 theta1b=angZG-angZC1b
500 if theta1b<0 then theta1b=180+theta1b
510 R=RL: X=XL: Z=Zo2: ABSZ=ABSZL: theta=theta2a
520 gosub 1200
530 phi2a=phi
535 if phi>0 then phi2a=phi-180
540 theta=theta2b
550 gosub 1200
560 phi2b=phi
```

```
565 if phi>0 then phi2b=phi-180
570 R=RG: X=XG: z=Zo1: ABSZ=ABSZG: theta=-theta1a
580 gosub 1200
590 phi1a=-phi
595 if phi1a>0 then phi1a=-phi-180
600 theta=-theta1b
610 gosub 1200
620 phi1b=-phi
625 if phi1b>0 then phi1b=-phi-180
630 phita=phi2a+phi1a: phitb=phi2b+phi1b
631 'print phi2a, phi1a
632 'print phi2b, phi1b
640 theta2a=(cint(theta2a*100))/100: theta2b=(cint(theta2b*100))/100
650 theta1a=(cint(theta1a*100))/100: theta1b=(cint(theta1b*100))/100
660 phita=(cint(phita*100))/100: phitb=(cint(phitb*100))/100            'total phase shift
670 print: print
680 Print "The two solutions in degrees are:"
690 print
800 print "     length of     length of     total phase "
810 print "     input line    output line      shift    "
820 print using "        ####.##";theta2a,theta1a,phita
830 print using "        ####.##";theta2b,theta1b,phitb

985 end
990 print: print
995 print "No solution is possible."
999 end


1000 H=R^2+X^2+Z^2
1010 SWR = (H + sqr(H^ 2 - 4 * (R^2) * (Z^2) + .0000001))/(2 * R*Z)
1020 cen = ((SWR^2+1)/(2*SWR))*Z              'center of SWR circle
1030 rho = ((SWR^2-1)/(2*SWR))*Z              'radius of the SWR circle
1040 K=R^2+X^2-Z^2
1050 ang=(57.29578)*(atn((K+sqr(K^2+4*(X^2)*(Z^2)+.0000001))/(2*(X+.0000001)*Z)))       'angle 1051'
of the impedance
1060 IF ang<0 then ang=180+ang
1070 ABSZ=sqr(R^2+X^2)
1071 'print SWR, ang
1090 return
1100 K=R^2+X^2-Z^2
1120 ang=(57.29578)*(atn((K+sqr(K^2+4*(X^2)*(Z^2)+.0000001))/(2*(X+.0000001)*Z)))
1130 IF ang<0 then ang=180+ang
1131 'print R, X, Z, ang
1140 return
1200 phi=-(57.29578)*atn((Z*R*tan(theta/57.29578))/(ABSZ^2+Z*X*tan(theta/57.29578)))  'phase shift
1211 'print R, X, Z, ABSZ, theta
1250 return
```

Another algorithmic method of solving the generalized problem is to find the transmission line length needed to transform the input and output impedances to pure resistances. For $Z=R+jX$, the lengths are given by

$$\frac{X}{SR - Z_0}$$

where S is either the SWR or its reciprocal. The problem of a series line transformer working between two pure resistances may then be solved and the line lengths required to transform the terminal impedances added or subtracted from this solution as appropriate. A Smith chart or R-X diagram should be used to correctly combine the various line lengths, as the operation can otherwise become confusing.

Phase Shift in Series Line Transformers

Note that the Basic program solves for the phase shift of the transformer. This parameter may be needed when using such transformers in driving antennas or in other phase sensitive applications. The phase shift of the input line is given by

$$\tan \Phi_2 = -\frac{Z_{o1} R_L \tan \Theta_2}{|Z_L|^2 + Z_{o1} X_L \tan \Theta_2},$$

and the phase shift in the output line is

$$\tan \Phi_1 = +\frac{Z_{o1} R_G \tan(-\Theta_1)}{|Z_G|^2 + Z_{o1} X_G \tan(-\Theta_1)}.$$

The total phase shift is the sum of the phase shifts of the input and output lines. The sign changes account for the fact that line 1 is being traversed away from the generator and towards the load. If the phase shift of either line is positive, it should be added to -180 degrees to obtain the correct value before adding to obtain the total phase shift.

The Series Section Transformer as a Four terminal Network

The series section transmission line transformer can be considered as a four-terminal network made up of two symmetric four terminal networks--the two line sections--in cascade (1). The resulting network is unsymmetric, with different characteristic impedances at the two terminal pairs. The network parameters are given by

$$Z_{01}^2 = Z_1^2 \frac{(Z_1 T_1 + Z_2 T_2)(Z_2 - Z_1 T_1 T_2)}{(Z_1 T_2 + Z_2 T_1)(Z_1 - Z_2 T_1 T_2)},$$

$$Z_{02}^2 = Z_2^2 \frac{(Z_1 T_1 + Z_2 T_2)(Z_1 - Z_2 T_1 T_2)}{(Z_1 T_2 + Z_2 T_1)(Z_2 - Z_1 T_1 T_2)},$$

$$\tanh^2 \gamma = -\frac{(Z_1 T_1 + Z_2 T_2)(Z_1 T_2 + Z_2 T_1)}{(Z_1 - Z_2 T_1 T_2)(Z_2 - Z_1 T_1 T_2)},$$

where T_i is $\tan\Theta_i$ and γ is the complex propagation coefficient, $\alpha + j\beta$. Z_{01} and Z_{02} are the characteristic impedances at the terminal pairs representing the terminals of lines 1 and 2.

The two characteristic impedances may be either real or imaginary For cases where the tangents are both positive or both negative the characteristic impedances are imaginary for

$$\frac{Z_1}{Z_2} < T_1 T_2 < \frac{Z_2}{Z_1},$$

where $Z_2 > Z_1$. Otherwise the inequalities are reversed. If the characteristic impedances are imaginary, the hyperbolic tangent of the propagation constant is real and if the characteristic impedances are real, the hyperbolic tangent is imaginary.

The network phase shift is given by:

For Z_{0i} real

$$\tan \varphi = - \frac{\frac{Z_{02}}{|Z_L|^2} R_L \tan \beta}{1 + \frac{Z_{02}}{|Z_L|^2} X_L \tan \beta}$$

For Z_{0i} imaginary, same sign

$$\tan \varphi = - \frac{\frac{Z_{02}}{|Z_L|^2} R_L \tanh \alpha}{+ \frac{Z_{02}}{|Z_L|^2} X_L \tanh \alpha}$$

For Z_{0i} imaginary, opposite sign

$$\tan \varphi = - \frac{\frac{Z_{02}}{|Z_L|^2} R_L}{\tanh \alpha + \frac{Z_{02}}{|Z_L|^2} X_L}$$

Z_{02} and X_i are magmitudes in these expressions. Z_{02} is considered to be the characteristic impedance at the terminals connected to the load.

If a network with a real characteristic impedance is matched to a resistive load, the phase shift is given by the value of the imaginary part of the propagation constant, β. Thus for an alternating line transformer matching 75 Ohms resistive to 50 Ohms resistive with 50 and 75 Ohm transmission lines, $Z_{02} = 75$ and Tanhy = jtanβ = j1.77953, $\beta = 60.666$ degrees. This network is equivalent to a 75 Ohm transmission line 60.666 degrees long followed by an ideal step down transformer with a 1.5:1 turns ratio.

References

(1) Weller. A. E. "Properties of Four-Terminal DissipationlessNetworks", unpublished manuscript, 1993

(2) Shone, A. B. and Wharton, W.,"Wide-Band Coaxial Transformers Using Solid Dielectric Cables", Electronic Engineering, April 1962, pp 252-255

(3) Regier, F. A., "Impedance Matching with a Series Transmission Line Section", Proc IEEE, Vol 59 July 1971, pp1133-1134: See Also Regier, Electronic Engineering, August 1973, pp 33-34 and Regier, QST July 1978, pp 14-16